Computational Economics (2025) 65:1319–1339
https://doi.org/10.1007/s10614-024-10597-2

Brazilian Selic Rate Forecasting with Deep Neural Networks

Rodrigo Moreira[1] · **Larissa Ferreira Rodrigues Moreira**[1,2] · **Flávio de Oliveira Silva**[2,3]

Accepted: 27 March 2024 / Published online: 15 April 2024

Abstract
Artificial intelligence has shortened edges in many areas, especially the economy, to support long-term and accurate forecasting of financial indicators. Traditional statistical methods perform poorly compared to those based on artificial intelligence, which can achieve higher rates even with high-dimensional datasets. This method still needs evolution and studies. In emerging countries, decision-makers and investors must follow the basic interest rate, such as in Brazil, with a Special System of Settlement and Custody (Selic). Prior works used deep neural networks (DNNs) for forecasting time series economic indicators such as interest rates, inflation, and the stock market. However, there is no empirical evaluation of the prediction models for the Selic interest rate, especially the impact of training time and the optimization of hyperparameters. In this paper, we shed light on these issues and evaluate, through a fair comparison, the use of DNNs models for Selic time series forecasting. Our results demonstrate the potential of DNNs with an error rate above 0.00219 and training time above 84.28 s. Our findings open up opportunities for further investigations toward real-time interest rate forecasting, facilitating more reliable and timely forecasting of interest rates for decision-makers and investors.

Keywords Economics · Selic rate · Artificial intelligence · Forecasting · DNNs · Optimization

✉ Larissa Ferreira Rodrigues Moreira
larissa.f.rodrigues@ufv.br

Rodrigo Moreira
rodrigo@ufv.br

Flávio de Oliveira Silva
flavio@ufu.br; flavio@di.uminho.pt

1 Institute of Exact and Technological Sciences (IEP), Federal University of Viçosa (UFV), Rio Paranaíba, Minas Gerais, Brazil

2 Faculty of Computing, Federal University of Uberlândia (UFU), Uberlândia, Minas Gerais, Brazil

3 Department of Informatics, School of Engineering, University of Minho, Braga, Portugal

1 Introduction

Computational tools such as Artificial Intelligence have been cutting edges in many areas, such as medical, natural, and economic sciences (Solgi et al., 2021; Albuquerque et al., 2022; Mohan et al., 2022; Rodrigues Moreira et al., 2023). Especially in economics, time series prediction is still difficult and is being widely studied in machine learning (Masini et al., 2021; Benidis et al., 2022; Kumbure et al., 2022).

In machine learning, several algorithms are specialists in clustering, prediction, or regression tasks; among them, we have the Deep Neural Networks (DNNs) that mimic how the brain learns (Wang et al., 2017; Zou et al., 2019). In this context, the community is evolving and applying such Machine Learning (ML) techniques in the economy to increase the accuracy of these models (Garcia et al., 2017) whereas ML is achieving relatively better performances than classical time series forecasting techniques (Sezer et al., 2020).

Inflation, growth, and interest rates are essential to measure the economic performance of emerging countries and guide public policies and private business planning. Accurate predictions from these economic indicators contribute to the economy's confidence, reducing uncertainty with insights into a long-term horizon (Aras & Lisboa, 2022; Bravo & El Mekkaoui, 2022; Medeiros et al., 2021; Basher & Sadorsky, 2022).

Many strategies based on statistical methods have attempted to predict interest rates since the 1930s (Fisher, 1930). Especially in emerging countries, which experience high-interest rates and volatile inflation, robust strategies for forecasting trends in these indicators must be used. Brazil conquered hyperinflation in 1994, so monetary policies periodically adjust the Special System of Settlement and Custody (Selic) interest rate, the primary interest rate of Brazil's Central Bank. Equivalent to the Federal Reserve (Fed) interest rate, the Brazilian Selic rate is used as a reference for daily financing rates secured by government bonds (Garcia et al., 2017).

Robust mathematical methods, such as Cox-Ingersoll-Ross (CIR) (Altman & Bland, 1983), can accurately predict time series (Orlando et al., 2020a, b; Bufalo & Orlando, 2023), particularly in the context of interest rates. However, there are still opportunities for research on using Deep Neural Networks (DNNs) and their behavior in this task. Therefore, this study aims to clarify and compare primarily the capacity of DNNs for time series forecasting. The advantages of DNNs in time-series forecasting include the flexibility to adapt to different data types: univariate or multivariate, discrete or continuous, stationary or non-stationary (Kumbure et al., 2022). They also offer the ability to use different architectures, input transformations, output models, and loss functions according to the specific context, forecast horizon, and relevant metrics. In addition, DNNs can incorporate long-term memory and temporal context using recurrent networks such as LSTM and GRU, overcoming the limitations of long-range dependencies and the disappearing gradient problem. These networks have proven effective in empirical studies by comparing their predictive performance with that of traditional methods in various scenarios and time series.

Linear and non-linear statistical models are widely used methods for dealing with time series forecasting (Bufalo et al., 2022). Also, sophisticated techniques such as autoregressive (AR) models and the Philips curve can predict time series. However, empirical evidence of the performance of DNNs compared to traditional forecasting techniques is lacking, especially in interest rate prediction in emerging countries (Siami-Namini et al., 2018).

This work proposes an experimental evaluation of traditional statistical models and DNNs, focusing primarily on understanding how DNNs behave in financial time series forecasting. We believe this paper is the first to propose a fair comparison of Brazil's interest rate (Selic) forecasting techniques while using hyperparameter optimization of DNNs models to achieve the best possible results. Hyperparameter optimization aims to find a high-performing model by searching for the best hyperparameter values in a search space (Bergstra et al., 2011).

In a nutshell, our main contributions are as follows: (1) a comparative study of DNNs for time series forecasting—while (Garcia et al., 2017; Livieris et al., 2020) used traditional forecasting models; (2) a training time assessment of DNNs for time series forecasting—although (Lu et al., 2020, 2021; Rezaei et al., 2021) assessed only forecasting performance metrics; (3) a hyperparameter optimization evaluation and its impact on DNNs models in economic forecasting—while (Richardson et al., 2021) evaluated hyperparameter optimization in traditional forecasting machine learning approaches.

In addition, this paper is organized as follows: Sect. 2 dives into related works and contrasts them against our contribution. In Sect. 3, we present our method, explain basic concepts of time series forecasting and machine learning and the protocol evaluation used in this paper. Section 4 presents our evaluation results and the interpretations and inferences from those results. Finally, in Sect. 5, we conclude and point out some future works and research challenges.

2 Related Work

Traditionally, statistical and mathematical tools were applied to time series to employ accurate and timely predictions. However, machine learning methods seemed suitable for this task. They have been applied in many fields such as energy forecasting, pandemic diseases, CO_2 emissions, and economy forecasting (Chou & Tran, 2018; Sezer et al., 2020; Wang et al., 2022; Llanos et al., 2022; Vargas et al., 2022). In this section, we describe some literature efforts to shed light on how machine learning improves time series forecasting.

Garcia et al. (2017) applied high-dimensional and machine learning models to estimate real-time forecasts of the Brazilian Extended National Consumer Price Index (IPCA) inflation. They used an enhanced Least Absolute Shrinkage and Selection Operator (LASSO) approach to perform short-term forecasting. Our paper addressed short-term forecasting with Deep Neural Network (DNN) achieving a lower error rate.

Livieris et al. (2020) proposes and evaluates the performance of an Recurrent Neural Network (RNN) to predict the price of gold. They propose combining convolutional and pooling layers to develop input data features and exploit generated features by Long Short-Term Memory (LSTM). Using the dataset from Yahoo Finance, the data were divided, and training and testing were set from Jan 2014 to Dec 2017. The test data ranged from Jan 2018 to Apr 2018. Carried experiments showcased that the proposed model performed well considering Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). Our paper goes beyond comparing much DNNs with baseline statistical regression using a basic interest rate dataset.

Vidal and Kristjanpoller (2020) innovated with a hybrid solution of RNN and Convolutional Neural Network (CNN) for time series prediction of the gold price. Among the steps of the proposed method, we consider transforming a time series into an image using the Gramian Angular Fields (GAF) method. To evaluate the proposed model, they used CNN VGG16 (Szegedy et al., 2015) and considered loss metrics. They validated their method using the London Bullion Market Association (LBMA) dataset. Our work goes further by performing DNN training, hyperparameters tuning, and performance comparison of relevant prediction models.

Similarly, Lu et al. (2020) proposed and evaluated the combination of RNN and CNN in the time series prediction of the Shanghai Composite Index (SSE). The proposal is to use CNN to extract features and LSTM to learn about features in the training process. The combination of these technologies proved to be promising compared to particular applications, considering the metrics established in state-of-the-art. Unlike the authors, we consider Recurrent Neural Networks (RNNs) using hyperparameter optimization for time series prediction.

Lu et al. (2021) brings a proposal that combines CNN, Bi-Directional Long Short-Term Memory (BiLSTM), and Attention Mechanism (AM) to predict the closing price of the Shanghai Composite Index (SSE) considering the metrics MAE, RMSE and R^2 that inform how well the predictor variables can explain the variation in the response variable. Among the existing prediction methods, the authors' proposal achieves the best performance metrics in prediction.

Rezaei et al. (2021) combined CNN, LSTM with Complete Empirical Ensemble Mode Decomposition (CEEMD), which can reduce the complexity of data series and convert and normalize data by removing we are. The CEEMD model decomposes the time series into different frequency spectra dealing with non-linear and non-stationary time series. They evaluate this proposed method with many asset indexes and measure the model performance using well-known state-of-the-art metrics. Our work exploits the use of CNN for interest rate prediction using hyperparameter optimization.

Livieris et al. (2021) proposed and evaluated the Multiple-Input Cryptocurrency Deep Learning Model (MICDL) method, which merges independent parts of the time series for achieving accurate and reliable prediction using a concatenation layer. They evaluate the proposed methods considering BTC, ETH, and XRP cryptocurrencies from the CoinMarketCap dataset. In our work, we exploit CNN to predict the Brazilian Interest Rate, known as Selic and assessed the hyperparameter optimization and their impact on model performance.

Richardson et al. (2021) compared five ML-based forecasting predictors with the auto-regressive model in a Gross Domestic Product (GDP) dataset of New

Zealand. Also, they noticed that fine-tuned ML models excel in statistical-based approaches. They used the Reserve Bank of New Zealand (RBNZ) dataset to validate the model. Similarly, Babii et al. (2022) tried to estimate the US GDP through a sparse-group LASSO method that supports high-dimensional time series data even then sampled at different frequencies.

In addition to studies based on DNNs, other studies are based on more advanced forecasting approaches. Studies undertaken by Orlando et al. (2020a, 2020b) and Orlando and Bufalo (2021) improved the CIR model for forecasting time series in finance. Recently, Bufalo and Orlando (2023) utilized the CIR model in the context of the European tourism industry, and Guerrero et al. (2023) applied in the context of the COVID-19 pandemic. Despite the relative simplicity of the CIR model in terms of data and complexity, it demonstrates robustness during financial turmoil.

Stoop et al. (2022) and Orlando et al. (2022) utilized deterministic-based models in the financial markets, revealing previously undetected deterministic patterns. Furthermore, they discovered that a broad range of empirical data, including equities and emerging markets, can be modeled using deterministic models during periods of financial turmoil.

We summarized our literature review through Table 1, detailing the contributions of each state-of-the-art approach. In the column Financial Asset/Rate/Index, we consider the qualitative description of the target forecast. Some works used machine learning to forecast the Gold price (Livieris et al., 2020), others criptocyrrencies (Livieris et al., 2021). In the column Forecast Model, we describe which methods were applied to forecast predictions; some approaches used CNN combined with other techniques (Lu et al., 2021; Rezaei et al., 2021). The column Performance Metric relates to the mathematical tool to asses the forecasting performance of the machine learning models. Many approaches used Mean Square Error (MSE), RMSE and MAE (Lu et al., 2020, 2021). Also, we used the column Dataset to describe which data were used to extract data for model evaluation.

3 Proposed Comparison

The Selic rate is an instrument for the Brazilian Central Bank to apply monetary policy, being the reference rate for the other rates. In this paper, we exploited the use of DNNs for forecasting the Brazilian Selic Rate. We contrast the use of DNNs with their optimized parameters with traditional statistical regression techniques. Also, we assess the training time of each DNN.

Figure 1 refers to the methodology used in this paper. Reading it from left to right, we noticed that the dataset feeds the evaluation setup hosted on compute resources. The evaluation setup has both classes of forecasting algorithms: regression and DNN-based. The ML models are subjected to hyperparameter tuning to achieve better forecasting performance. Both reports feed the performance comparison considering the MAE and RMSE, and the tunned models are publicly published.

Table 1 Short state-of-the-art survey

Approach	Financial Asset/Rate/Index	Forecast model	Performance metric	Dataset	Hyperparameters tunning
Garcia et al. (2017)	Brazilian IPCA	Lasso and FOCUS	RMSE and MAE	Brazilian Central Bank	NA
Bhattacharjee and Bhattacharja (2019)	Tesla Asset	MMA, AR, ARIMA, SARIMA, Lasso, Ridge, KNN, SVM, RF, SLP, MLP, RL, and LSTM	MSE, MAE, RMSE, and R^2	Tesla Dataset	×
Livieris et al. (2020)	Gold	RNN	MAE, RMSE, ACC, AUC, SEM, and SPE	Yahoo! Finance	×
Vidal and Kristjanpoller (2020)	Gold	RNN + CNN	MSE, Improvement, and MCS	LBMA	×
Lu et al. (2020)	Shanghai Composite Index (SSE)	RNN + CNN	MAE, RMSE, and R^2	Yahoo! Finance	×
Lu et al. (2021)	Shanghai Composite Index (SSE)	CNN + BiLSTM + AM	MAE, RMSE, and R^2	Yahoo! Finance	×
Rezaei et al. (2021)	S &P500/Down Jones/DAX/Nikkei225	CEEMD + CNN + LSTM	MSE, MAE, and RMSE	Yahoo! Finance	×
Livieris et al. (2021)	Cryptocurrencies	MICDL	MAE, RMSE, and R^2	CoinMarketCap	×
Richardson et al. (2021)	GDP	SVM, Lsboost, Lasso, ElasticNet, and Ridge	RMSE	RBNZ	✓
Babii et al. (2022)	GDP	Lasso, Ridge, and ElasticNet	RMSE	US FED	×
Orlando et al. (2020a)	Interest Rate: EUR	Vasicek and CIR	MSE and MAE	EUR Interest Rates	NA
Orlando et al. (2020b)	Interest Rate: EUR and US Dollar	Vasicek and CIR	RMSE and R^2	EUR and US Dollar Interest Rates	NA
Orlando and Bufalo (2021)	Interest Rate: EUR, USD, JPY, and CHF	CIR and HW	RMSE	EUR, USD, JPY, and CHF Interest Rates	NA
Stoop et al. (2022)	Android's market position	Rulkov map	MAE	Android's market competition data	NA

Table 1 (continued)

Approach	Financial Asset/Rate/Index	Forecast model	Performance metric	Dataset	Hyperparameters tunning
Orlando et al. (2022)	Interest Rate: EUR	Rulkov map	MAE	Financial Stress Index STLFSI2	NA
Bufalo and Orlando (2023)	Rate: Tourism	VAR, BVAR, TVP, TFT, and EC-ADLM	RMSE, MAE, and Mean Absolute Percentage Error (MAPE)	Eurostat	NA
Guerrero et al. (2023)	Rate: COVID-19	CIR and ARIMAX	RMSE	COVID-19 in the city of Jaén (Spain)	NA
Our approach	Brazilian Selic Rate	DNN	MAE and RMSE	Brazilian Central Bank	✓

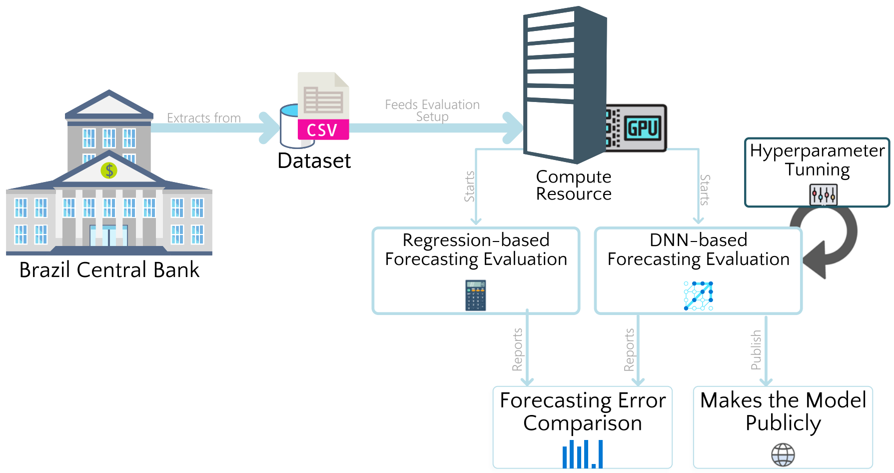


Fig. 1 Our evaluation approach

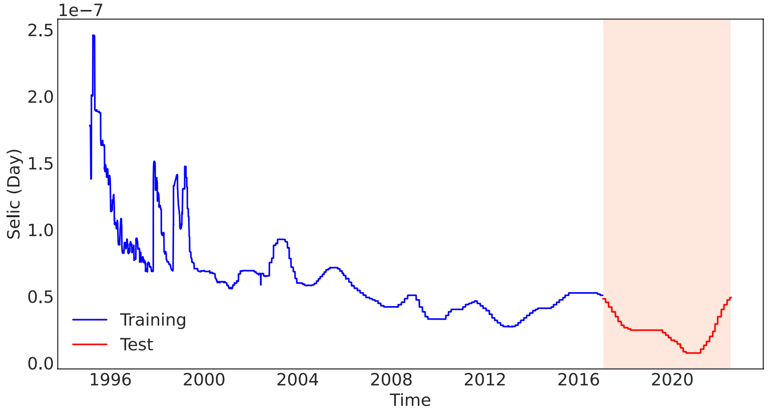


Fig. 2 Selic dataset splitted into training and test

3.1 Dataset

We extracted the dataset of the Central Bank of Brazil[1] that registers the basic interest rate since 1986, containing more than 9000 data. We consider the Selic rate records from the database after 1996 when the Selic rate records started to be recorded daily. We evaluate the forecasting performance of five (5) DNNs over the dataset using hyperparameter optimization.

Also, we split the dataset by a proportion of 80% for the training set and 20% for the test set. Figure 2 represents the time series and its division between

[1] Available in: https://dadosabertos.bcb.gov.br/dataset/11-taxa-de-juros-Selic.

training and test sets. Our dataset is one-dimensional and non-stationary. In addition, the main feature of the dataset is the daily record of the basic interest rate (Selic), represented by a decimal number.

3.2 Statistical Forecasting Methods

Our work evaluated two approaches to time series forecasting: one using DNNs and the other using traditional statistical models as a baseline. The prediction models are based on basic statistics, which are simple and univariate classes.

A simple regression model is a linear relationship between the forecast variable y and the prediction variable x. Univariate regression models attempt to model and predict a variable using its past information. In our evaluation, we consider the model that is based on the Moving Average (MA), represented by Eq. 1:

$$X_t = \mu + A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2}, -\ldots - \theta_q A_{t-q} \tag{1}$$

Where, X_t represents the time series, μ the mean of the series and A_{t-i} are the noises and $\theta_1 ..., \theta_q$ are the model parameters. We also consider the model based on the Exponential Moving Average (EMA) as a baseline in forecasting the Selic rate represented by Eq. 2. This model uses exponentially decreasing weights for each period.

$$EMA = (K \times (C - P)) + P \tag{2}$$

Where C represents the current value, P the previous EMA periods and K the exponential smoothing constant. Using basic forecasting models to compare the suitability of Convolutional Neural Networks (CNNs) for this task.

The basic statistical models have a little training time, but the forecast performance could be poorly evaluated. This evaluation supports choosing the better model by considering the performance and training time.

The CIR model is a stochastic model that describes the evolution of interest rates over time (Orlando et al., 2022; Bufalo & Orlando, 2023). It is based on the assumption that interest rates are driven by a single factor, which is the level of the short-term interest rate. The model uses a square-root diffusion process, which ensures that interest rates are always non-negative and mean-reverting. The model can be expressed by the following differential Eq. 3.

$$dr_t = a(b - r_t)dt + \sigma \sqrt{r_t} dW_t \tag{3}$$

Where r_t represents the instantaneous interest rate at time t, a is the speed of mean reversion, b is the long-term mean, σ is the volatility parameter, dW_t is the Wiener process increment, enables us to forecast interest rates by simulating future rate paths based on these parameters. The model provides insights into interest rate dynamics by capturing mean reversion tendencies and volatility effects.

3.3 DNN Forecasting Models

Due to their non-linear relationship, models based on DNNs have been successfully utilized in time series forecasting problems. In this work, we evaluate five (5) models and measure their performance, considering statistical methods as the baseline. We also measured the training and prediction time for each of the models. We optimize hyperparameters to obtain the best possible results in the training phase.

We choose prediction models based on two ML categories: RNN and CNN. As a variant of RNN, we use the model LSTM, a network capable of using the cyclic connection and updating the current state based on past states and current inputs. In addition, we use CNNs for time series forecasting. CNNs are commonly used for object detection but have recently been applied in time series (Chauhan & Singh, 2018). The models used are described below.

- *ResNet*: is a deep residual network that uses residual shortcut connection between the consecutive convolution layers to overcome the gradient vanishing effect (He et al., 2016). We evaluated the ResNet architecture proposed by Wang et al. (2017) to deal with time series. It comprises three residual blocks with three convolutional kernels whose output is added to the residual block's input and then fed into the next layer.
- *ResCNN*: is a deep residual network with a convolutional layer for time-series classification. It performs the convolution operation into three 1-D kernels with the sizes of 8, 5, 3 and without striding. Also, it removes the pooling operation, and a global average pooling is used instead of fully connected to alleviate overfitting (Zou et al., 2019).
- *LSTM:* is a RNN able to learn order dependency in non-linear sequence prediction problems. LSTM's main feature is remembering information for a long time. Thus, the memory cell replaces traditional neurons' hidden layers and forms the LSTM network's core. The recurring cell of LSTM remembers values at arbitrary time intervals, and the flow of information input and output of the cell is regulated by input, output, and forget gates (Hochreiter & Schmidhuber, 1997).
- *InceptionTime*: is an ensemble of five deep learning models for time-series classification, and each one considers multiple Inception modules to explore multiscale representations from data. In addition, an Inception module applies multiple filters simultaneously to an input time series (Ismail Fawaz et al., 2020).
- *Omni-Scale 1D-CNN (OS-CNN)*: adopts 1-D CNN, namely OS-block, to exploit the local features of time series and the relationships among the data. In addition, the OS-block uses a set of prime numbers as the kernel size, and the kernel sizes for the last layer are defined as 1 and 2 (Tang et al., 2020).

Table 2 shows the internal structure of each DNN, the description of the number of layers, the total parameters followed by the input and output features. Input features refer to the windowing of the time series that feeds DNN into the training process.

Table 2 DNNs experiment details

Model	Number of layers	Total parameters	Input features	Output features
ResCNN	27	256,130	100	1
ResNet	43	478,337	100	1
InceptionTime	48	388,481	128	1
LSTM	6	41,301	100	1
OS-CNN	49	484,924	100	1

Table 3 Hyperparameter Search Space

Batch Size	Learning Rate	Epochs	Patience	Optimizer
[16, 32, 64, 128]	[0.01, 0.001, 0.0001]	[20, 50, 100]	[5, 10]	[*Adam*, *SGD*]

3.4 Hyperparameter Optimization

The DNNs contain numerous parameters, leading to manual configuration and unsatisfactory results. To overcome this limitation, we applied a variant of Bayesian optimization called Tree-structured Parzen Estimator (TPE) to optimize the hyperparameters (Bergstra et al., 2011) automatically. Five major hyperparameters of the models were tuned: batch size, learning rate, epochs, patience, and optimizer. The details of the hyperparameter search space are presented in Table 3.

The hyperparameter optimization process is an early stop to save computational resources and training time. The Optimizer is a function that modifies the weights of the neural network. Our approach considers the Batch Size, which refers to the number of examples used at each iteration in the training phase. The Learning Rate determines the step size for each iteration in the loss function minimization process. The epoch parameter refers to the number of completed training steps over the entire dataset. The patient refers to the number of epochs without improvement to be considered in the early stagesop.

In addition, we employed an optimization process for the CIR model parameters to enhance the time-series prediction performance. Our optimization estimates the parameters using Ordinary Least Squares (OLS) and then refines these estimates using Maximum Likelihood Estimation (MLE). The CIR model is a stochastic process that models interest rates and assumes positive ones. The optimization process involves using the SciPy library's functions for least-squares fitting and optimization. The final output was the optimized parameters of the CIR model.

3.5 Evaluation Metrics

We used two evaluation functions to measure the forecasting task performance. The RMSE represents the standard deviations of the predictions, being it possible to check how far the predicted points are from the regression line. Its representation is according to Eq. 4.

$$RMSE(y, \hat{y}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{4}$$

Where y represents the real value and $\hat{y}$ the predicted value, verifying the model's performance given the difference between the predicted and the real.

In addition, we use MAE as a metric to evaluate the models; its representation is according to Eq. 5.

$$MAE(y, \hat{y}) = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i| \tag{5}$$

The forecasting performance metric MAE refers to the magnitude of difference between the prediction and the values measured in the prediction.

We ran the experiments on an Intel Core i5-4430 @3.00Ghz hardware, containing 32 GB of RAM and an Nvidia GTX 1080 Ti GPU. Aiming at reproducibility, our code is available and the results are in the open-source repository.[2]

All experiments were programmed using Python 3.8. The DNNs evaluated in our paper were implemented using the Tsai library with PyTorch and Fastai. To calculate model quality assessment metrics, we use scikit-learn. The hyperparameter optimization algorithm TPE was drawn from the hyperopt. Finally, the charts were generated using Matplotlib.

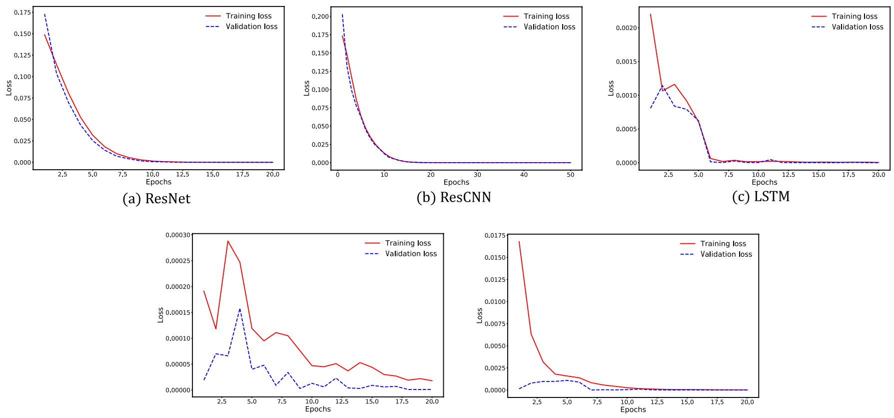
4 Results and Discussion

Firstly, we measured the model performance by assessing the Loss metric in the training and validation phase according to Fig. 3. As we can notice in all CNNs, there is an indication of a decrease in loss as long as training epochs advance, leading us to admit that models are suitable to handle Selic predictions.

We performed experiments to validate the suitability of CNNs in the time series forecasting task. We present the performance of CNN in Fig. 4, where it is possible to recognize the generalization capacity of the model, suggesting learning when the predicted curve follows the real tendency. Comparing the ResCNN curve with its peers leads us to conclude that it had the worst performance in learning since the curves are visibly disjoint. Note that when the real values suggest a drop in the Selic between 2020 and 2021, the predicted values of ResCNN (Fig. 4b) could not follow this trend.

As can be seen from Fig. 5, we employed the CIR model, which followed the same trend as the actual interest rate but had a lower variability and a higher lag.

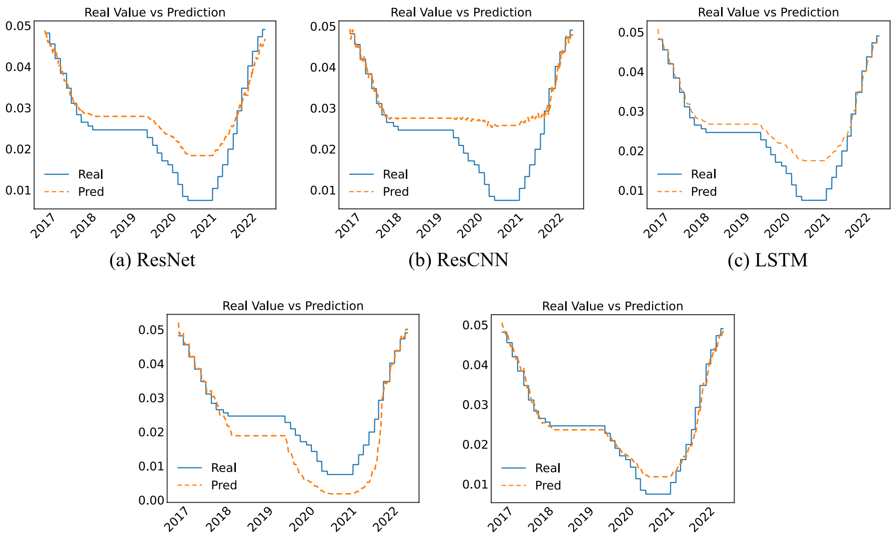
[2] Available in: https://github.com/romoreira/Selic-TSPrediction



(a) ResNet (b) ResCNN (c) LSTM

(d) InceptionTime (e) OS-CNN

Fig. 3 Loss measurement for each deep neural network



(a) ResNet (b) ResCNN (c) LSTM

(d) InceptionTime (e) OS-CNN

Fig. 4 Forecasting error measurement for each deep neural network

In our experiments, the CIR model had a mean error of approximately 0.01 and a coefficient of determination of approximately 0.8. This means that the CIR model explains about 80% of the variation in the interest rate but has a margin of error of 1 percentage point. The CIR model, despite being explainable, customizable, and robust, achieved moderate quality in predicting the interest rate.

The generalization capacity of the models represented by Fig. 4 varies between CNNs. We used initial 80% samples of the time series for the CNN training process, while the remaining 20% of the time series was used to evaluate the

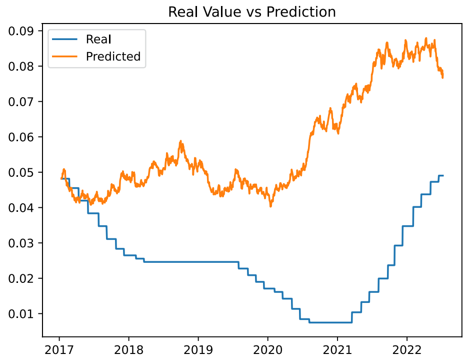


Fig. 5 Forecasting error measurement of the CIR model

Table 4 Forecasting Performance Comparison

Model	RMSE	MAE	Training Time (seconds)
MA	0.16972	0.02880	–
EMA	0.22622	0.05117	–
CIR	0.03781	0.03308	–
ResCNN	0.00894	0.00635	116.61
ResNet	0.00577	0.00475	98.96
InceptionTime	0.00648	0.00529	219.92
OS-CNN	**0.00219**	**0.00163**	84.28
LSTM	0.00480	0.00357	69.70

Best results were highlighted as bold

performance of the models considering their pairs. After, we measured the error rate of each CNN using MAE and RMSE.

To measure the quality of the predictions, we use the functions of MAE and RMSE that measure the error through the difference between the real values and the predictions. Thus, the smaller the error, the more accurate the model tends to be, leading to more accurate forecasting.

To compare the performance of the model, we built Table 4 that presents the OS-CNN as the one capable of forecasting with the smallest RMSE and MAE among the CNNs considered in our experiment. In addition, we measured the training time of each model and found that CNN LSTM required the least time to train and adjust their weights. Even though LSTM took the least amount of time, the results suggest that its performance in forecasting is unsatisfactory compared to its peers.

Also, the regression-based methods performed poorly compared to those using DNN according to Table 4. Thus, the EMA for the Selic dataset outperformed the MA considering the metrics RMSE and MAE. For the sake of comparison, we did not consider the training times for the statistical models because they are negligible.

We consider the models and compare their performance in the Selic forecasting task without the previous hyperparameter adjustment task. In addition, we evaluated

Table 5 Forecasting performance without hyperparameter optimization

Model	RMSE	MAE
ResCNN	0.09280	0.09084
ResNet	0.01958	0.01288
InceptionTime	0.01886	0.01316
OS-CNN	**0.00968**	**0.00688**
LSTM	0.32213	0.32192

Best results were highlighted as bold

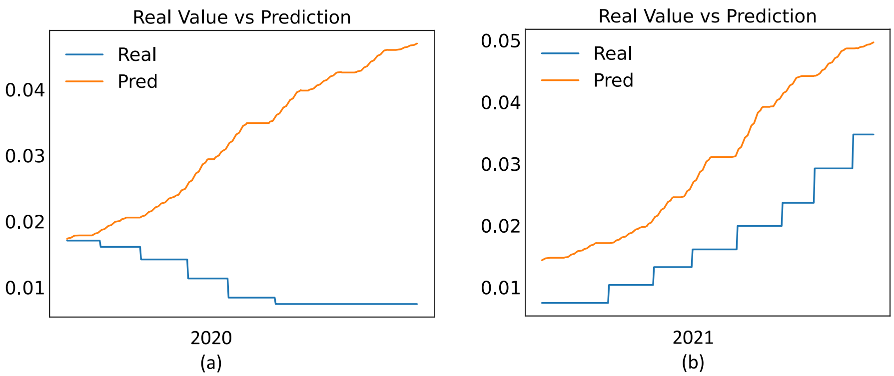


Fig. 6 Sliced performance over the entire period from 2020 to 2021

the pertinence of hyperparameter adjustment. Thus, Table 5 demonstrates that CNNs without hyperparameter adjustments obtain inferior results compared to models trained considering good hyperparameters.

For both training and validation, ResCNN led to the worst performance metrics for forecasting, making it necessary to reconsider its use in Selic time series forecasting. Furthermore, it is necessary to consider not only the forecasting performance but also the training time of the models. Even though LSTM does not have the best accuracy, the model requires the least training time, about 17.2% less than OS-CNN. In this sense, the time series forecasting with the Selic dataset seemed suitable for the time series forecasting with CNNs.

We carried out a more in-depth analysis to assess the robustness of the model in scenarios of financial turmoil and public health factors such as COVID-19. We then chose the DNN model that performed best in our evaluations, OmniScaleCNN, and sliced the test predictions into two years, 2020 and 2021, and measured the distance between the actual and predicted values using the RMSE and MAE metrics.

As can be seen in Fig. 6a, the year 2020 alone represents a challenging scenario for CNNs because the model predicts an upward trend in the juicing rate, while the actual figures suggest a downward trend. However, in Fig. 6b, 2021 alone represents a scenario in which the predictions were correct, even though 2021 represents a period of inflection in the trends due to various factors. Table 6 lists the prediction errors for two-time slices of the time series.

Table 6 In-Depht Analysis of Sliced Years of Interest Rate

Sliced Year	RMSE	MAE
2020	0.01276	0.01249
2021	0.00952	0.00927

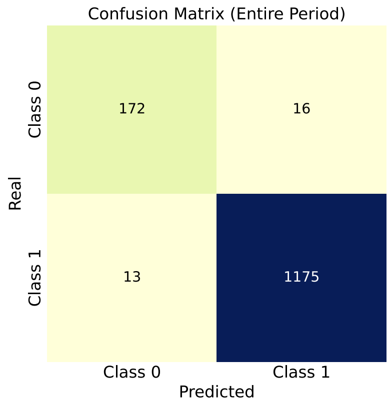


Fig. 7 Error measurement for the whole period

We believe that the performance of DNNs is related to their number of parameters. However, such correlation is not positive for training time since OS-CNN has more parameters and requires less training time than ResCNN, which has fewer parameters.

With this finding, we evaluate technologies that can be used in interest rate predictions and support decision-makers. Thus, our observations showcased CNNs open the opportunity to build real-time forecasting tools with high accuracy rates.

We conducted an analysis of the DNN that performed best for the forecasting task, for which we constructed a confusion matrix. The method used to build the confusion matrix involved discretizing values: those with a distance between the actual and predicted values greater than 1% belonged to Class 0, while those with less than 1% belonged to Class 1. Thus, we denote two classes: Class 0 as "Out of forecasting Interval" and Class 1 as "Inside forecasting interval".

The confusion matrix Fig. 7 reveals the evaluation of the best-scored DNN model distinguishes between two classes: "Out of forecasting Interval" (Class 0) and "Inside forecasting Interval" (Class 1). It correctly predicted 172 instances as "Out of forecasting Interval" and 1,175 instances as "Inside forecasting Interval". However, it misclassified 16 instances as "Inside forecasting Interval" when they were "Out of forecasting Interval" and 13 instances as "Out of forecasting Interval" when they were inside the defined interval. Overall, the model demonstrates strong accuracy in predicting within the interval but shows some misclassification outside this range.

In-depth analysis for 2020, Fig. 8 reveals reliable predictive performance: correctly identifying 144 instances and accurately predicting 40 cases. However, it misclassifies 67 instances as "Out of forecasting Interval", indicating limitations in

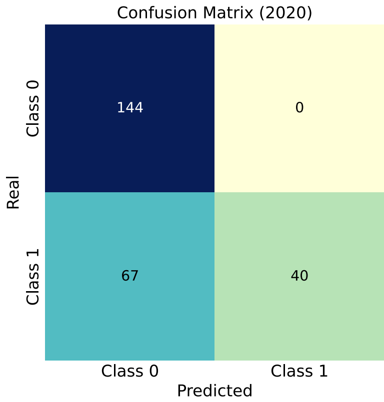


Fig. 8 Error measurement for 2020

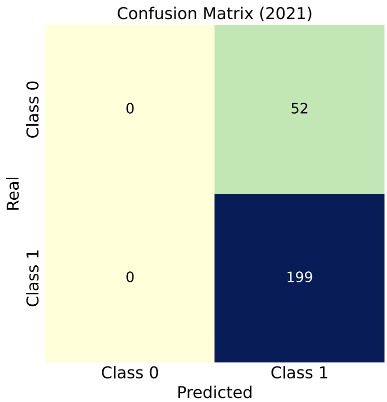


Fig. 9 Error measurement for 2021

accurately forecasting certain events within a specific timeframe. The analysis of 2021 in the confusion matrix (Fig. 9) indicates a notable trend: correctly predicting 199 instances within the forecasting interval but failing to capture any instances outside it, showcasing a limitation in forecasting accuracy beyond the defined range.

Our study aimed to assess the efficacy of DNNs in time series forecasting. Although there are uncomplicated and dependable mathematical techniques available for this task, our findings indicate that DNNs can still be utilized effectively because of their adaptability to the specific problem at hand.

5 Concluding Remarks

Although there are robust mathematical methods, our study focused on analyzing the behavior of DNNs for time series forecasting, especially for the interest rate. The Selic rate is an instrument the Brazilian Central Bank uses to define the basic interest rate. We found that DNNs outperform statistical methods in the prediction process of these time series. Our short state-of-the-art survey points out that DNN techniques were not explored for time series forecasting of Brazilian interest rates.

Well-trained and fine-tuned models support the implementation of applications and devices for real-time forecasting. We carried out a fair comparison DNNs, basic statistics, and a robust model CIR was evaluated in our scenario. We found that DNNs performed better despite training time consumption. Furthermore, we found that DNNs mutually diverged in forecasting performance and training time owing to their internal structure. Also, we observe that fine-tuning DNN hyperparameters leads to more accurate forecasting models without changing model size.

Our study was limited to evaluating the forecasting performance of the Brazilian interest rate (Selic). At the same time, future research will include assessing other economic indicators, such as Bitcoin price, stock market, and government bonds. We also are looking forward to investigating the variations in the CIR model and its effectiveness for the Selic interest rate. In addition, we intend to evaluate the impact of training DNNs models on forecasting high-dimensional datasets, assessing model interpretability, and building training methods that consider multiple time series. Additionally, it is crucial to investigate research opportunities related to assessing external interference, such as politics and geopolitics, on the price behavior of assets and financial indicators.

Acknowledgements The authors acknowledge the financial support of the Brazilian National Council for Scientific and Technological Development (CNPq), grant #421944/2021-8. Also, this study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

Author Contributions All authors contributed to the study's conception and design. Material preparation, data collection, and analysis were performed by R. M. and L. F. R. M. The writing process involved contributions from all authors. All authors wrote, read, and approved the final manuscript.

Funding F.D.S has received support by Brazilian National Council for Scientific and Technological Development (CNPq), grant #421944/2021-8. L. F. R. M. has received support by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

Declarations

Conflict of interest The authors have no conflicts of interest to declare relevant to this article's content.

References

Albuquerque, P. C., Cajueiro, D. O., & Rossi, M. D. (2022). Machine learning models for forecasting power electricity consumption using a high dimensional dataset. *Expert Systems with Applications, 187*, 115917. https://doi.org/10.1016/j.eswa.2021.115917

Altman, D. G., & Bland, J. M. (1983). Measurement in medicine: The analysis of method comparison studies. *Journal of the Royal Statistical Society: Series D (The Statistician), 32*(3), 307–317. https://doi.org/10.2307/2987937

Aras, S., & Lisboa, P. J. (2022). Explainable inflation forecasts by machine learning models. *Expert Systems with Applications, 207*, 117982. https://doi.org/10.1016/j.eswa.2022.117982

Babii, A., Ghysels, E., & Striaukas, J. (2022). Machine learning time series regressions with an application to nowcasting. *Journal of Business & Economic Statistics, 40*(3), 1094–1106. https://doi.org/10.1080/07350015.2021.1899933

Basher, S. A., & Sadorsky, P. (2022). Forecasting bitcoin price direction with random forests: How important are interest rates, inflation, and market volatility? *Machine Learning with Applications, 9*, 100355. https://doi.org/10.1016/j.mlwa.2022.100355

Benidis, K., Rangapuram, S. S., Flunkert, V., Wang, Y., Maddix, D., Turkmen, C., & Januschowski, T. (2022). Deep learning for time series forecasting: Tutorial and literature survey. *ACM Computing Surveys*. https://doi.org/10.1145/3533382

Bergstra, J., Bardenet, R., Bengio, Y., & Kégl, B. (2011). Algorithms for hyper-parameter optimization. In J. Shawe-Taylor, R. Zemel, P. Bartlett, F. Pereira, & K. Weinberger (Eds.), *Advances in neural information processing systems* (Vol. 24, pp. 1–9). Curran Associates Inc.

Bhattacharjee, I., & Bhattacharja, P. (2019). Stock price prediction: A comparative study between traditional statistical approach and machine learning approach. In *2019 4th International conference on electrical information and communication technology (EICT)* (pp. 1–6).

Bravo, J. M., & El Mekkaoui, N. (2022). Short-term CPI inflation forecasting: Probing with model combinations. In A. Rocha, H. Adeli, G. Dzemyda, & F. Moreira (Eds.), *Information systems and technologies* (pp. 564–578). Springer.

Bufalo, M., Liseo, B., & Orlando, G. (2022). Forecasting portfolio returns with skew-geometric Brownian motions. *Applied Stochastic Models in Business and Industry, 38*(4), 620–650. https://doi.org/10.1002/asmb.2678

Bufalo, M., & Orlando, G. (2023). Improved tourism demand forecasting with CIR# model: A case study of disrupted data patterns in Italy. *Tourism Review*. https://doi.org/10.1108/TR-04-2023-0230

Chauhan, N. K., & Singh, K. (2018). A review on conventional machine learning versus deep learning. In *2018 International conference on computing, power and communication technologies (GUCON)* (pp. 347–352).

Chou, J.-S., & Tran, D.-S. (2018). Forecasting energy consumption time series using machine learning techniques based on usage patterns of residential householders. *Energy, 165*, 709–726. https://doi.org/10.1016/j.energy.2018.09.144

Fisher, I. (1930). *Theory of interest: As determined by impatience to spend income and opportunity to invest it*. Augustusm Kelly Publishers.

Garcia, M. G., Medeiros, M. C., & Vasconcelos, G. F. (2017). Real-time inflation forecasting with high-dimensional models: The case of Brazil. *International Journal of Forecasting, 33*(3), 679–693. https://doi.org/10.1016/j.ijforecast.2017.02.002

Guerrero, J., del Carmen Galiano, M., & Orlando, G. (2023). Modeling COVID-19 pandemic with financial markets models: The case of Jaén (Spain). *Mathematical Biosciences and Engineering, 20*(5), 9080–9100. https://doi.org/10.3934/mbe.2023399

He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In *2016 IEEE conference on computer vision and pattern recognition (CVPR)* (pp. 770–778).

Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural Computation, 9*(8), 1735–1780. https://doi.org/10.1162/neco.1997.9.8.1735

Ismail Fawaz, H., Lucas, B., Forestier, G., Pelletier, C., Schmidt, D. F., Weber, J., & Petitjean, F. (2020). Inception time: Finding AlexNet for time series classification. *Data Mining and Knowledge Discovery, 34*(6), 1936–1962. https://doi.org/10.1007/s10618-020-00710-y

Kumbure, M. M., Lohrmann, C., Luukka, P., & Porras, J. (2022). Machine learning techniques and data for stock market forecasting: A literature review. *Expert Systems with Applications, 197*, 116659. https://doi.org/10.1016/j.eswa.2022.116659

Livieris, I. E., Kiriakidou, N., Stavroyiannis, S., & Pintelas, P. (2021). An advanced cnn-lstm model for cryptocurrency forecasting. *Electronics*. https://doi.org/10.3390/electronics10030287

Livieris, I. E., Pintelas, E., & Pintelas, P. (2020). A CNN-LSTM model for gold price time-series forecasting. *Neural Computing and Applications, 32*(23), 17351–17360. https://doi.org/10.1007/s00521-020-04867-x

Llanos, C., Kristjanpoller, W., Michell, K., & Minutolo, M. C. (2022). Causal treatment effects in time series: CO2 emissions and energy consumption effect on GDP. *Energy, 249*, 123625. https://doi.org/10.1016/j.energy.2022.123625

Lu, W., Li, J., Li, Y., Sun, A., & Wang, J. (2020). A CNN-LSTM-based model to forecast stock prices. *Complexity, 2020*, 6622927. https://doi.org/10.1155/2020/6622927

Lu, W., Li, J., Wang, J., & Qin, L. (2021). A CNN-BiLSTM-AM method for stock price prediction. *Neural Computing and Applications, 33*(10), 4741–4753. https://doi.org/10.1007/s00521-020-05532-z

Masini, R. P., Medeiros, M. C., & Mendes, E. F. (2021). Machine learning advances for time series forecasting. *Journal of Economic Surveys*. https://doi.org/10.1111/joes.12429

Medeiros, M. C., Vasconcelos, G. F. R., Veiga, Á., & Zilberman, E. (2021). Forecasting inflation in a data-rich environment: The benefits of machine learning methods. *Journal of Business & Economic Statistics, 39*(1), 98–119. https://doi.org/10.1080/07350015.2019.1637745

Mohan, S., Solanki, A. K., Taluja, H. K., Anuradha, & Singh, A. (2022). Predicting the impact of the third wave of covid-19 in India using hybrid statistical machine learning models: A time series forecasting and sentiment analysis approach. *Computers in Biology and Medicine, 144*, 105354. https://doi.org/10.1016/j.compbiomed.2022.105354

Orlando, G., & Bufalo, M. (2021). Interest rates forecasting: Between Hull and White and the CIR#-How to make a single-factor model work. *Journal of Forecasting, 40*(8), 1566–1580. https://doi.org/10.1002/for.2783

Orlando, G., Bufalo, M., & Stoop, R. (2022). Financial markets' deterministic aspects modeled by a low-dimensional equation. *Scientific Reports, 12*(1), 1693. https://doi.org/10.1038/s41598-022-05765-z

Orlando, G., Mininni, R. M., & Bufalo, M. (2020a). Forecasting interest rates through Vasicek and CIR models: A partitioning approach. *Journal of Forecasting, 39*(4), 569–579. https://doi.org/10.1002/for.2642

Orlando, G., Mininni, R. M., & Bufalo, M. (2020b). A new approach to forecast market interest rates through the CIR model. *Studies in Economics and Finance, 37*(2), 267–292. https://doi.org/10.1108/SEF-03-2019-0116

Rezaei, H., Faaljou, H., & Mansourfar, G. (2021). Stock price prediction using deep learning and frequency decomposition. *Expert Systems with Applications, 169*, 114332. https://doi.org/10.1016/j.eswa.2020.114332

Richardson, A., van Florenstein Mulder, T., & Vehbi, T. (2021). Nowcasting GDP using machine-learning algorithms: A real-time assessment. *International Journal of Forecasting, 37*(2), 941–948. https://doi.org/10.1016/j.ijforecast.2020.10.005

Rodrigues Moreira, L. F., Moreira, R., Travençolo, B. A. N., & Backes, A. R. (2023). An artificial intelligence-as-a-service architecture for deep learning model embodiment on low-cost devices: A case study of COVID-19 diagnosis. *Applied Soft Computing, 134*, 110014. https://doi.org/10.1016/j.asoc.2023.110014

Sezer, O. B., Gudelek, M. U., & Ozbayoglu, A. M. (2020). Financial time series forecasting with deep learning: A systematic literature review: 2005–2019. *Applied Soft Computing, 90*, 106181. https://doi.org/10.1016/j.asoc.2020.106181

Siami-Namini, S., Tavakoli, N., & Siami Namin, A. (2018). A comparison of ARIMA and LSTM in forecasting time series. In *2018 17th IEEE international conference on machine learning and applications (ICMLA)* (pp. 1394–1401).

Solgi, R., Loáiciga, H. A., & Kram, M. (2021). Long short-term memory neural network (LSTM-NN) for aquifer level time series forecasting using in-situ piezometric observations. *Journal of Hydrology, 601*, 126800. https://doi.org/10.1016/j.jhydrol.2021.126800

Stoop, R., Orlando, G., Bufalo, M., & Della Rossa, F. (2022). Exploiting deterministic features in apparently stochastic data. *Scientific Reports, 12*(1), 19843. https://doi.org/10.1038/s41598-022-23212-x

Szegedy, C., Liu, W., Jia, Y., Sermanet, P., Reed, S., Anguelov, D., & Rabinovich, A. (2015). Going deeper with convolutions. In *2015 IEEE conference on computer vision and pattern recognition (CVPR)* (pp. 1–9).

Tang, W., Long, G., Liu, L., Zhou, T., Jiang, J., & Blumenstein, M. (2020). Rethinking 1D-CNN for time series classification: A stronger baseline. CoRR arXiv:abs/2002.10061

Vargas, G., Silvestre, L., Rigo Júnior, L., & Rocha, H. (2022). B3 stock price prediction using LSTM neural networks and sentiment analysis. *IEEE Latin America Transactions, 20*(7), 1067–1074.

Vidal, A., & Kristjanpoller, W. (2020). Gold volatility prediction using a CNN-LSTM approach. *Expert Systems with Applications, 157*, 113481. https://doi.org/10.1016/j.eswa.2020.113481

Wang, Z., Yan, W., & Oates, T. (2017). Time series classification from scratch with deep neural networks: A strong baseline. In *2017 International joint conference on neural networks (IJCNN)* (pp. 1578–1585).

Wang, Y., Yan, Z., Wang, D., Yang, M., Li, Z., Gong, X., & Wang, Y. (2022). Prediction and analysis of COVID-19 daily new cases and cumulative cases: Times series forecasting and machine learning models. *BMC Infectious Diseases, 22*(1), 495. https://doi.org/10.1186/s12879-022-07472-6

Zou, X., Wang, Z., Li, Q., & Sheng, W. (2019). Integration of residual network and convolutional neural network along with various activation functions and global pooling for time series classification. *Neurocomputing, 367*, 39–45. https://doi.org/10.1016/j.neucom.2019.08.023