Towards a linear algebra semantics for query languages

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Abstract

There has been renewed interest on **columnar** database systems.

Row-storage abandoned in favor of the **1-attribute / 1-file** scheme.

Traditional vendors of row-store systems (e.g. Oracle, Microsoft) have added **column-oriented features** to their product lineups.

WHY?

This talk will address the advantages of **columnar** storage from a **formal semantics** point of view.

A **columnar semantics** for SQL will be sketched based on (typed) **linear algebra**.

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A **columnar semantics** for SQL will be sketched based on (typed) **linear algebra**.

Context

Linear algebra (LA) in formal semantics...

LA proving essential elsewhere, eg:

- Physics
- Econometrics
- ...

Algebraic approach to **quantitative** formal methods, e.g.

- *A study of risk-aware program transformation* (Murta, Oliveira: SCP 2015)
- *Relational Algebra for "Just Good Enough" Hardware* (RAMiCS 2014)

on handling **risk** (of failure) in programming:

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Context

FP7-ICT 619606

About the project:

> *“(...)* ***queries*** *[identifying] facts of interest take hours, days, or weeks, whereas business processes demand today shorter cycles.*

Project motto: *lean* big data!

However — **what** are we actually **leaning**?

What is, after all, a **query**?

Back to basics (SQL)

There are **jobs**:

```
create table jobs (
  j_code   char (15) not null,
  j_desc   char (50),
  j_salary decimal (15, 2) not null);
```

j_code	**j_desc**	**j_salary**
Pr	*Programmer*	*1000*
SA	*System Analyst*	*1100*
GL	*Group Leader*	*1333*

Back to basics

There are **employees**:

```
create table empl (
  e_id       integer not null,
  e_job      char (15) not null,
  e_name     char (15),
  e_branch   char (15) not null,
  e_country  char (15) not null);
```

e_id	e_job	e_name	e_branch	e_country
1	*Pr*	*Mary*	*Mobile*	*UK*
2	*Pr*	*John*	*Web*	*UK*
3	*GL*	*Charles*	*Mobile*	*UK*
4	*SA*	*Ana*	*Web*	*PT*
5	*Pr*	*Manuel*	*Web*	*PT*

Query

Monthly salary total per country / branch:

select *e_country*, *e_branch*, *sum* (*j_salary*)
  from *empl*, *jobs*
  where $j\_code = e\_job$
  group by *e_country*, *e_branch*
  order by *e_country*;

sqlite3:

```
PT|Web|2100
UK|Mobile|2333
UK|Web|1000
```

Query

Impact of

insert into "jobs" **values** ('SA', 'System Admin', 1000);

that is, *j_code* no longer a key.

sqlite3:

```
PT|Web|3100
UK|Mobile|2333
UK|Web|1000
```

Fine — so *SA* is taken as a kind of "multi-job".

But — where are these quantitative **semantics** specified?

Standard semantics

Given in English:

> *"The result of evaluating a query-specification can be explained in terms of a multi-step algorithm. The order of [the 7] steps in this algorithm follows the mandatory order of the clauses (FROM, WHERE, and so on) of the SELECT statement"*

Cf. pages 71-73 of

> *X/Open CAE Specification Data Management: Structured Query Language (SQL) Version 2 March 1996, X/Open Company Limited*

7 steps

1. *For each table-reference that is a joined-table, conceptually join the tables (...) to form a single table*
2. *Form a Cartesian product of all the table-references (...)*
3. *Eliminate all rows that do not satisfy the search-condition in the WHERE clause.*
4. *Arrange the resulting rows into groups (...)*
   - *If there is a GROUP BY clause specifying grouping columns, then form groups so that all rows within each group have equal values for the grouping columns (...)*
5. *If there is a HAVING clause, eliminate all groups that do not satisfy its search-condition (...)*
6. *Generate result rows based on the result columns specified by the select-list (...)*
7. *In the case of SELECT DISTINCT, eliminate duplicate rows from the result (...)*

Background

Join operator — ok, well defined in Codd's relation algebra.

However,

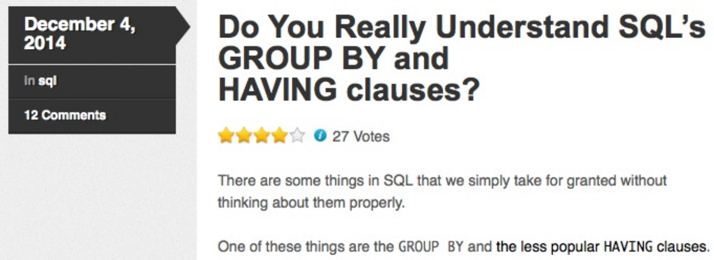
> *[...] relational DBMS were never intended to provide the very powerful functions for data synthesis, analysis and consolidation that is being defined as multi-dimensional data analysis.*
>
> *E.F.Codd* [1]

> *[...] expressing roll-up, and cross-tab queries with conventional SQL is daunting. [...] GROUP BY is an unusual relational operator [...]*
>
> *J. Gray et al* [2]

[1] Providing OLAP to User-Analysts: An IT Mandate (1998)

[2] Data Cube: A Relational Aggregation Operator Generalizing Group-By, Cross-Tab, and Sub-Totals (1997)

Background



December 4, 2014

In sql

12 Comments

Do You Really Understand SQL's GROUP BY and HAVING clauses?

27 Votes

There are some things in SQL that we simply take for granted without thinking about them properly.

One of these things are the GROUP BY and the less popular HAVING clauses.

[http://blog.jooq.org/2014/12/04/do-you-really-understand-sqls-group-by-and-having-clauses/]

Background

Why these shortcomings / questions ?

> *While* **relation algebra** *"à la Codd" [works] well for qualitative data science [it is] rather clumsy in handling the quantitative side [...] we propose to solve this problem by suggesting* **linear algebra** *(LA) as an alternative suiting both sides [...]*
>
> *H. Macedo, J. Oliveira* [3]

[3] A linear algebra approach to OLAP (2015)

Formalizing SQL data aggregation

VLDB'87, among other research:

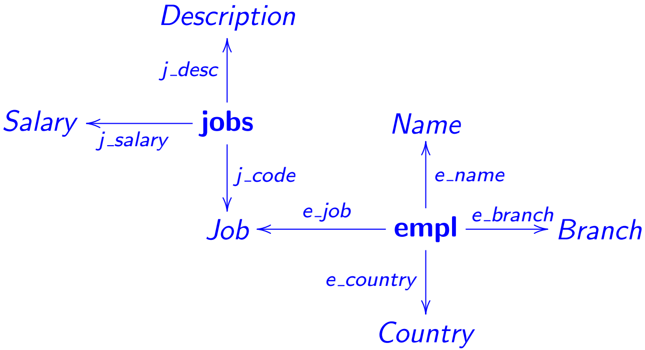
SQL Query	Calculus Expression
SELECT $f_1,\ldots,f_l$ **FROM** $r_1(v_1),\ldots,r_n(v_n)$ **WHERE** P_w	$(f'_1,\ldots,f'_l)(* : r_1(v_1),\ldots,r_n(v_n) : P_w)$
SELECT $t_1,\ldots,t_l \ (\neq f)$ **FROM** $r_1(v_1),\ldots,r_n(v_n)$ **WHERE** P_w	$(t_1,\ldots,t_l) : r_1(v_1),\ldots,r_n(v_n) : P_w$
SELECT $t_1,\ldots,t_l$ **FROM** $r_1(v_1),\ldots,r_n(v_n)$ **WHERE** P_w **GROUP BY** $v_{i_1}[A_{i_1}],\ldots,v_{i_k}[A_{i_k}]$ **HAVING** P_h	$(t'_1,\ldots,t'_l) : \alpha(v) : P'_h$ $\alpha = (\phi_{<(A_{i_1},\ldots,A_{i_k}),(f'_1,\ldots,f'_m)>}(* : r_1(v_1),\ldots,r_n(v_n) : P_w));$ $(t'_1,\ldots,t'_l,P'_h) = (t_1,\ldots,t_l,P_h)[f_i/v[k+i], v_{i_j}[A_{i_j}]/v[j]];$ $(f_1,\ldots,f_m$ aggregate functions in $t_1,\ldots,t_l, P_h)$

G. Bultzingsloewen [4]

[4] Translating and optimizing SQL queries having aggregates (1987)

"Star" diagrams

Entities (cf. tables) surrounded (placed at the center of) by their **attributes**:



Entities marked in bold.

Attribute types made explicit, linking entities to each other.

"Star" diagrams

What is the (formal) meaning of the **arrows** in the diagram?

There is one arrow per **attribute** — **column** in the database table.

Assigning meanings to the arrows amounts to formalizing a **columnar** approach to SQL.[5]

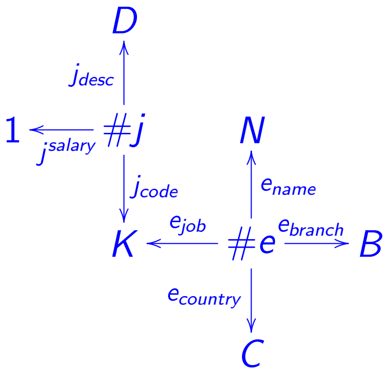
Let us do so using the **linear algebra of programming** (LAoP).[6]

---

[5] D. Abadi et al, *The Design and Implementation of Modern Column-Oriented Database Systems* (2012).

[6] J. Oliveira, *Towards a Linear Algebra of Programming* (2012).

Formal star-diagram in (**typed**) LAoP



- **Measures**:
  - *salary*

- **Types**:
  K — Job code
  D — Job description
  C — Country
  B — Branch
  N — Name
  $\#e$ — *empl* record nrs
  $\#j$ — *jobs* record nrs
- **Dimensions**:
  - *branch*
  - *name*
  - *country*
  - *job*
  - *desc*
  - *code*

Dimensions

Dimension attribute columns are captured by **bitmap** matrices:

e_{branch}	1	2	3	4	5
Mobile	1	0	1	0	0
Web	0	1	0	1	1

e_{job}	1	2	3	4	5
GL	0	0	1	0	0
Pr	1	1	0	0	1
SA	0	0	0	1	0

$e_{country}$	1	2	3	4	5
PT	0	0	0	1	1
UK	1	1	1	0	0

j_{desc}	1	2	3
Group Leader	0	0	1
Programmer	1	0	0
System Analyst	0	1	0

j_{code}	1	2	3
GL	0	0	1
Pr	1	0	0
SA	0	1	0

Meaning of bitmap **matrix** t_d, for d a dimension of table t:

$$v \ t_d \ i = 1 \quad \Leftrightarrow \quad t[i].d = v \tag{1}$$

Measures

However — main difference wrt. **relation algebra** — we won't build

j_{salary}	1	2	3
1000	1	0	0
1100	0	1	0
1333	0	0	1

but rather the **row vector** $j^{salary} : \#j \rightarrow 1$ which "internalizes" the quantitative information:

j^{salary}	1	2	3
1	1000	1100	1333

Summary:

Measures are *vectors*, **dimensions** are *matrices*.

Linear algebra

Matrices are **arrows** — e.g. $B \xleftarrow{M} C$ — cf. **categories** of matrices.

Matrix **multiplication**, given matrices $B \xleftarrow{M} C \xleftarrow{N} A$:

$$b\,(M \cdot N)\,a \;=\; \langle \sum c \;::\; (b\;M\;c) \times (c\;N\;a) \rangle \tag{2}$$

Matrix **converse**:

$$c\;M^{\circ}\;b \;=\; b\;M\;c \tag{3}$$

Functions are (special cases of Boolean) matrices:

$$y\;f\;x \;=\; \left\{ \begin{array}{l} 1 \textbf{ if } y = f\;x \\ 0 \; \mathit{otherwise} \end{array} \right. \tag{4}$$

The **identity** matrix $id : A \to A$ is the unit of composition.

Examples

$1 \xleftarrow{j^{salary}} \#j \xleftarrow{j^{\circ}_{code}} K$	Pr	SA	GL
1	1000	1100	1333

Calculation:

$$
\begin{array}{ll}
& 1\ (j^{salary} \cdot j^{\circ}_{code})\ k \\
= & \{\ \text{multiplication (2)}\ \} \\
& \langle \sum y\ ::\ (1\ j^{salary}\ y) \times (y\ j^{\circ}_{code}\ k) \rangle \\
= & \{\ \text{converse (3) ; vector } j^{salary}\ \} \\
& \langle \sum y\ ::\ (k\ j_{code}\ y) \times (j[y].salary) \rangle \\
= & \{\ \text{functions (4) ; quantifier notation (details soon)}\ \} \\
& \langle \sum y\ :\ k = j[y].code\ :\ j[y].salary \rangle
\end{array}
$$

$\square$

Examples

In case of

insert into "jobs" **values** ('SA', 'System Admin', 1000);

we get non-injective bitmap

j_{code}	1	2	3	4
GL	0	0	1	0
Pr	1	0	0	0
SA	0	**1**	0	**1**

and

j^{salary}	1	2	3	4
1	1000	1100	1333	1000

Therefore:

$1 \xleftarrow{j^{salary}} \#j \xleftarrow{j_{code}^{\circ}} K$	Pr	SA	GL
1	1000	**2100**	1333

Pointwise LAoP calculus

Given a **binary** predicate $p : B \times A \rightarrow Bool$, we denote by $[\![p]\!] : B \leftarrow A$ the Boolean matrix which encodes p, that is,

$$b\ [\![p]\!]\ a = \textbf{if}\ p\ (b, a)\ \textbf{then}\ 1\ \textbf{else}\ 0 \qquad (5)$$

In case of a **unary** predicate $q : A \rightarrow Bool$, $[\![q]\!] : 1 \leftarrow A$ is the Boolean vector such that:

$$1\ [\![q]\!]\ a = \textbf{if}\ q\ a\ \textbf{then}\ 1\ \textbf{else}\ 0 \qquad (6)$$

We often abbreviate

$$1\ [\![q]\!]\ a$$

by

$$q\ [a].$$

Pointwise LAoP calculus

Quantifier notation follows the Eindhoven style,

$$\langle \sum x \ : \ R : \ T \rangle$$

where R is a predicate (**range**) and T is a numeric term.

In case $T = B \times M$ where Boolean $B = [\![P]\!]$ encodes predicate P, we have the **trading rule**:

$$\langle \sum x \ : \ R : \ [\![P]\!] \times M \rangle \ = \ \langle \sum x \ : \ R \wedge P : \ M \rangle \qquad (7)$$

Thus

$$y(f \cdot N)x \quad = \quad \langle \sum z \ : \ y = f\,z : \ z\,N\,x \rangle \qquad (8)$$

$$y(g^\circ \cdot N \cdot f)x \quad = \quad (g\ y)\ N\ (f\ x) \qquad (9)$$

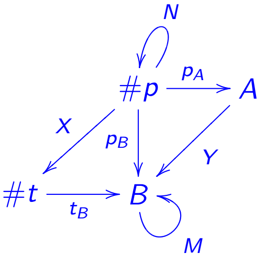
hold, where f and g are functions.

Joins and tabulations

Querying amounts to **following paths** in star diagrams.

The **meaning of a path** is obtained by *composing* (multiplying) the *matrices* involved.

Two particular such compositions deserve special reference, as they correspond to well-known operations in data processing:



- **Join**: $X = t_B^\circ \cdot M \cdot p_B$
- **Tabulation**: $Y = p_B \cdot N \cdot p_A^\circ$

M and N are whatever matrices of their type.

Simple Examples

Equi-join ($M = id$):

$j^{\circ}_{code} \cdot e_{job}$	1	2	3	4	5
1	1	1	0	0	1
2	0	0	0	1	0
3	0	0	1	0	0

Pointwise meaning: $j[y].code = e[x].job$ recall rule (9).

Counting tabulation ($N = id$):

$e_{country} \cdot e^{\circ}_{branch}$	Mobile	Web
PT	0	2
UK	2	1

Pointwise meaning: $\langle \sum k \ : \ y = e[k].country \wedge x = e[k].branch : \ 1 \rangle$ recall (8), for y a country, x a branch.

Columnar joins

Excerpt from Abadi et al[7]

For example, the figure below shows the results of a join of a column of size 5 with a column of size 4:

$$\begin{array}{|c|}\hline 42\\\hline 36\\\hline 42\\\hline 44\\\hline 38\\\hline\end{array} \bowtie \begin{array}{|c|}\hline 38\\\hline 42\\\hline 46\\\hline 36\\\hline\end{array} = \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline 5\\\hline\end{array}\ \begin{array}{|c|}\hline 2\\\hline 4\\\hline 2\\\hline 1\\\hline\end{array}$$

shows **columnar-join** "isomorphic" to our matrix joins:

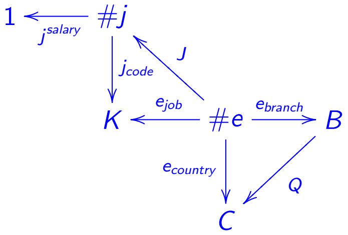
	1	*2*	*3*	*4*	*5*
1	*0*	*0*	*0*	*0*	*1*
2	*1*	*0*	*1*	*0*	*0*
3	*0*	*0*	*0*	*0*	*0*
4	*0*	*1*	*0*	*0*	*0*

[7] *The Design (..) of Modern Column-Oriented Database Systems* (2012).

Back to the starting SQL query

select
e_branch,
e_country,
sum (*j_salary*)
from *empl*, *jobs*
where $j\_code = e\_job$
group by
e_country,
e_branch
order by
e_country;

Minimal diagram accommodating query:



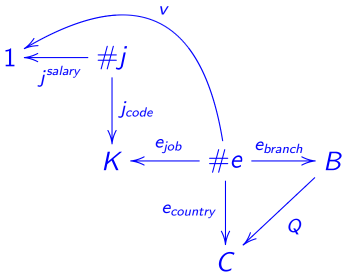
Clearly,

group by $\Rightarrow$ *tabulation* Q
where $\Rightarrow$ *join* J

Back to the starting SQL query

select
e_branch,
e_country,
sum (*j_salary*)
from *empl*, *jobs*
where *j_code* = *e_job*
group by
e_country,
e_branch
order by
e_country;

How do **salaries** get involved? We need a direct path from employees to (their) salaries,



extending the **where**-clause **join**:

$$v = j^{salary} \cdot j_{code}^{\circ} \cdot e_{job} \qquad (10)$$

Query = Group by + Join

The **group by** clause calls for a tabulation — but, how does vector

$v = j^{salary} \cdot j^{\circ}_{code} \cdot e_{job}$	1	2	3	4	5
1	1000	1000	1333	1100	1000

get into the place of N in the generic scheme?

Easy: every vector v can be turned into a **diagonal** matrix, e.g.

$v \triangledown id$	1	2	3	4	5
1	1000	0	0	0	0
2	0	1000	0	0	0
3	0	0	1333	0	0
4	0	0	0	1100	0
5	0	0	0	0	1000

and vice versa.

Khatri-Rao product

This diagonalization resorts to another LA operator, termed Khatri-Rao product ($M \triangledown N$) defined by

$$(b, c)\ (M \triangledown N)\ a \quad = \quad (b\ M\ a) \times (c\ N\ a) \tag{11}$$

Then:

$$\begin{array}{ll} & b\ (v \triangledown id)\ c \ = \ v\,[c] \times (b\ id\ c) \\ \Leftrightarrow & \quad \{\ \text{Khatri-Rao (11) ; function } id\ \} \\ & b\ (v \triangledown id)\ c \ = \ v\,[c] \times (b = c) \\ \Leftrightarrow & \quad \{\ \text{pointwise LAoP (5)}\ \} \\ & b\ (v \triangledown id)\ c \ = \ \textbf{if } b = c \textbf{ then } v\,[c] \textbf{ else } 0 \end{array}$$

i.e. non-zeros can only be found in the **diagonal**.

Linear algebra

Property of diagonal matrices:

$$(v \triangledown id) \cdot (u \triangledown id) = (v \times u) \triangledown id \tag{12}$$

where $M \times N$ is the matrix Hadamard product:

$$b\ (M \times N)\ a = (b\ M\ a) \times (b\ N\ a) \tag{13}$$

Moreover, for f a function, rule

$$f \triangledown v = f \cdot (v \triangledown id) \tag{14}$$

holds:

$$
\begin{array}{ll}
 & b\ (f \cdot (v \triangledown id))\ a \\
\Leftrightarrow & \quad \{ \text{ composition ; Khatri-Rao } \} \\
 & \langle \sum c\ ::\ (b\ f\ c) \times (v\ [a] \times (c\ id\ a)) \rangle \\
\Leftrightarrow & \quad \{ \text{ trading (7) ; cancel } \sum \text{ cf. } c = a \ \} \\
 & (b\ f\ a) \times v\ [a] \\
\Leftrightarrow & \quad \{ \text{ Khatri-Rao } \} \\
 & b\ (f \triangledown v)\ a
\end{array}
$$

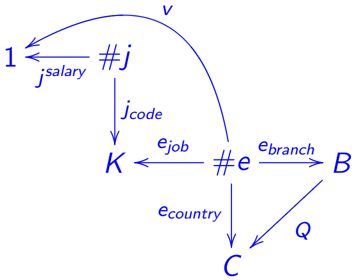
□

Query = Group by + Join

Query:

select
e_branch,
e_country,
sum (*j_salary*)
from *empl*, *jobs*
where *j_code* = *e_job*
group by
e_country,
e_branch
order by
e_country;

Diagram:



LA semantics:

$$Q = e_{country} \cdot (v \triangledown id) \cdot e_{branch}^{\circ} \qquad (15)$$

where $v = j^{salary} \cdot j_{code}^{\circ} \cdot e_{job}$

Pointwise semantics

Of vector v first:

$$
\begin{array}{ll}
& v\,[k] \\
= & \{\ \text{definition (10)}\ \} \\
& 1\,(j^{salary} \cdot j_{code}^{\circ} \cdot e_{job})\ k \\
= & \{\ \text{matrix multiplication (2)}\ \} \\
& \langle \sum\ i\ ::\ (1\ j^{salary}\ i) \times (i\ (j_{code}^{\circ} \cdot e_{job})\ k) \rangle \\
= & \{\ \text{trading rules (9) and (7)}\ \} \\
& \langle \sum\ i\ :\ j_{code}\ i = e_{job}\ k\ :\ (1\ j^{salary}\ i) \rangle \\
= & \{\ \text{pointwise notation conventions}\ \} \\
& \langle \sum\ i\ :\ j[i].code = e[k].job\ :\ j[i].salary \rangle \\
\Box &
\end{array}
$$

Pointwise semantics

Of the whole query:

$$c\ Q\ b$$

$$= \quad \{\ \text{definition (15) ; diagonal } v \triangledown id\ \}$$

$$\langle \sum\ k\ ::\ (c\ e_{country}\ k) \times (k\ (v \triangledown id)\ k) \times (k\ e^{\circ}_{branch}\ b)\rangle$$

$$\Leftrightarrow \quad \{\ \text{trading rule (7)}\ \}$$

$$c\ Q\ b = \langle \sum\ k\ :\ c = e_{country}\ k \wedge b = e_{branch}\ k :\ v\ [k]\rangle$$

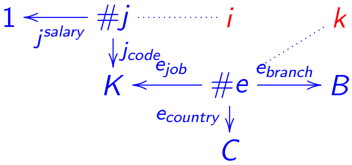
Putting both together:

$$query\ (c, b) = \sum k, i :$$
$$c = e[k].country \wedge b = e[k].branch \wedge j[i].code = e[k].job :$$
$$j[i].salary$$

Rest point :-)

Clearly:

- SQL is a **path-language**
- SQL is **pointfree** — see how the surface language **hides** the double-cursor i, k pointwise **for**-loop.



SQL tries to be as **pointfree** as **natural** language is so, compare

"there's no place like home"

with the (boring!)

$$\langle \forall\ p\ :\ p \in Place :\ p < home \rangle$$

(We don't **speak** using "cursors"...)

Simplification

LA semantics (15)

$$Q = e_{country} \cdot (v \triangledown id) \cdot e^{\circ}_{branch} \textbf{ where } v = j^{salary} \cdot j^{\circ}_{code} \cdot e_{job}$$

can be simplified into

$$Q = (e_{country} \triangledown v) \cdot e^{\circ}_{branch}$$

thanks to Khatri-Rao law (14). Note how matrix

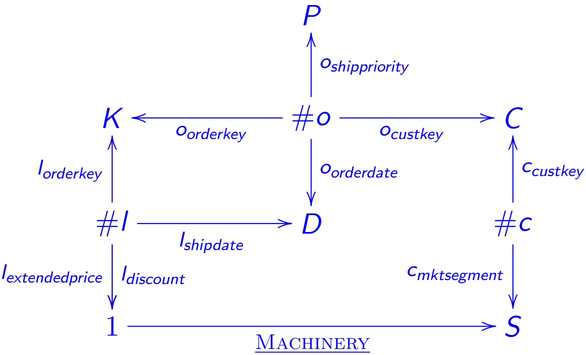
$e_{country} \triangledown v$	1	2	3	4	5
PT	0	0	0	1100	1000
UK	1000	1000	1333	0	0

nicely combines **qualitative** (functional) with **quantitative** information.

LA script for TPC-H query3

$query3 =$
select
$l\_orderkey, o\_orderdate, o\_shippriority;$
$sum\ (l\_extendedprice * (1 - l\_discount))\ as\ revenue$
from
$orders, customer, lineitem$
where
$c\_mktsegment = \texttt{'MACHINERY'}$
$and\ c\_custkey = o\_custkey$
$and\ l\_orderkey = o\_orderkey$
$and\ o\_orderdate < date\ \texttt{'1995-03-10'}$
$and\ l\_shipdate > date\ \texttt{'1995-03-10'}$
group by
$l\_orderkey, o\_orderdate, o\_shippriority$
order by
$revenue\ desc, o\_orderdate;$

Diagram for TPC-H query3



“Big-plan” **tabulation** again dictated by the **group by** clause:

$$Q = K \xleftarrow{l_{orderkey}} \#l \xleftarrow{X} \#o \xleftarrow{(o_{shippriority} \triangledown o_{shipdate})^{\circ}} P \times D$$

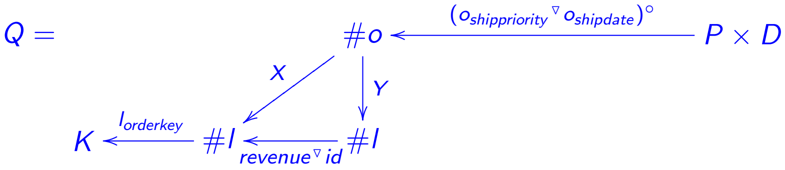
LA semantics for TPC-H query3

Data aggregation is performed over a derived vector

$$revenue = l_{extendedprice} \times (! - l_{discount}) \qquad (16)$$

where $! : \#l \to 1$ is the unique (constant) function of its type — a row vector wholly filled with ones.

We move on:



LA semantics for TPC-H query3

As expected, the link Y between the two tables is the join in the **where** clause:

$$
\begin{array}{ccccccc}
 & & & & \#o & \xleftarrow{(o_{shippriority} \triangledown o_{shipdate})^\circ} & P \times D \\
 & & & & \downarrow {\scriptstyle Y = (l_{orderkey})^\circ \cdot o_{orderkey}} & & \\
K & \xleftarrow{l_{orderkey}} & \#l & \xleftarrow[revenue \triangledown id]{} & \#l & &
\end{array}
$$

LA semantics for TPC-H query3

Moving on, clauses

```
o_orderdate < date '1995-03-10'
and l_shipdate > date '1995-03-10'
```

convert to vectors

$v : \#o \rightarrow 1$
$u : \#l \rightarrow 1$

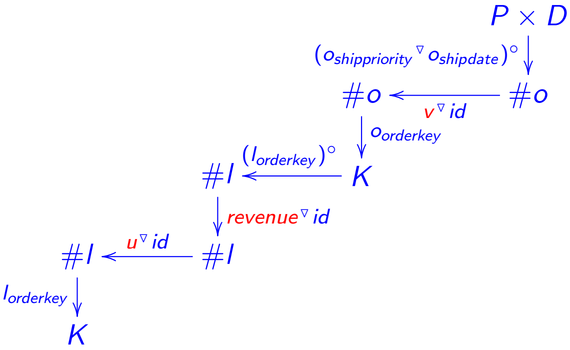
defined by

$$v\ [i] = [\![o[i].orderdate < \text{'1995-03-10'}]\!]$$
$$u\ [k] = [\![l[k].shipdate > \text{'1995-03-10'}]\!]$$

recall (6).

LA semantics for TPC-H query3

Altogether, thus far:



where $v\ [i] = [\![o[i].orderdate < \text{'1995-03-10'}]\!]$
and $u\ [k] = [\![l[k].shipdate > \text{'1995-03-10'}]\!]$

LA semantics for TPC-H query3

Finally, clauses

```
c_mktsegment = 'MACHINERY' and c_custkey = o_custkey
```

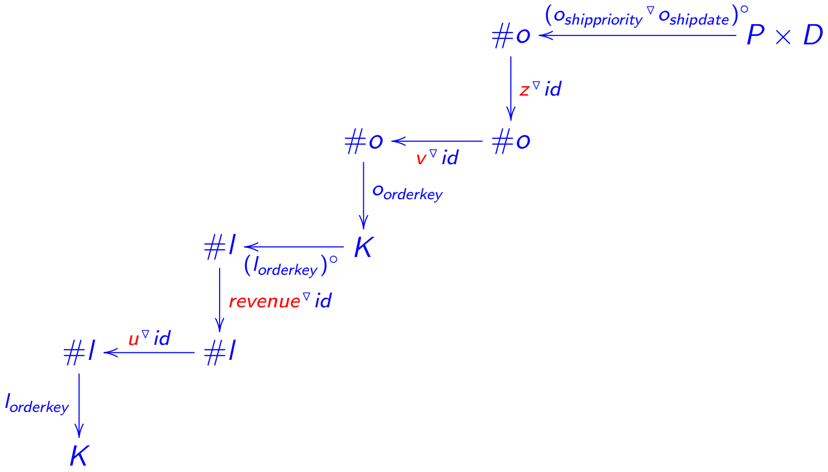
amount to Boolean path (vector)

$$z = 1 \xleftarrow{\textsc{Machinery}^\circ} S \xleftarrow{c_{mktsegment}} \#c \xleftarrow{c^\circ_{custkey}} C \xleftarrow{o_{custkey}} \#o$$

which **counts** how many customers exhibit the specified market segment:

$$z\,[k] = \langle \sum i \; : \; c[i].custkey = o[k].custkey \wedge c[i].mktsegment = MACHINERY \; : \; 1 \rangle$$

Query final path



Simplification of ("water fall") path

Thanks to LA laws:

$$Q3 = \quad \#o \xleftarrow{(o_{shippriority} \triangledown o_{shipdate})^{\circ}} P \times D$$

$$\#o \xrightarrow{o_{orderkey} \triangledown (v \times z)} K$$

$$\#l \xleftarrow{(l_{orderkey})^{\circ}} K$$

$$\#l \xrightarrow{(l_{orderkey}) \triangledown (revenue \times u)} K$$

Notice the same overall pattern: a **join** inside a **tabulation**.

Other simplifications possible, likely impacting on **performance** — in **what** sense ?

Divide and conquer

Block linear algebra enables **distributed** evaluation of query paths by "divide & conquer" laws for **all** operators involved, cf.

$$[A|B] \cdot \left[\frac{C}{D}\right] = A \cdot C + B \cdot D \tag{17}$$

$$\left[\frac{A}{B}\right]^{\circ} = [A^{\circ}|B^{\circ}] \tag{18}$$

and

$$[A|B] \triangledown [C|D] = [A \triangledown C|B \triangledown D] \tag{19}$$

$$[A|B] \times [C|D] = [A \times C|B \times D] \tag{20}$$

which generalize to **any finite** number of blocks.

Map-reduce

Overall path splits in two parts,

- Workload over table $\#o$:

$$\begin{array}{ccc} \#o & \xleftarrow{(o_{shippriority} \triangledown o_{shipdate})^{\circ}} & P \times D \\ \big\downarrow{\scriptstyle o_{orderkey} \triangledown (v \times z)} & & \\ K & & \end{array}$$

- Workload over table $\#l$:

$$\begin{array}{ccc} \#l & \xleftarrow[(l_{orderkey})^{\circ}]{} & K \\ {\scriptstyle (l_{orderkey})^{\triangledown}(revenue \times u)}\big\downarrow & & \\ K & & \end{array}$$

With n **machines**, each table is divided into n **slices**, each slice residing in its machine.

Map runs the two workloads on each machine, in **parallel**.

Reduce joins all machine-contributions together, then performing the final composition of the 2 paths.

Summary

Recall the X/Open CAE Specification:

> *"The result of evaluating a query-specification can be explained in terms of a multi-step algorithm. The order of [the 7] steps in this algorithm follows the mandatory order of the clauses (FROM, WHERE, and so on) of the SELECT statement"*

Our **evaluation order** is clearly different !

It is "demand driven" by the **group by** clause.

In theory, everything is **embarrassingly parallel**... but read this MSc dissertation [8] before getting too excited...

[8] R. Pontes, Benchmarking a Linear Algebra Approach to OLAP (2015)

Practical side

Future (practical) work:

- Define a **DSL** for the LA **path** language
- Mount a **map-reduce** interpreter for such a DSL running on a data-distributed environment
- Write a **compiler** mapping (a subset of) **SQL** to the DSL
- Enjoy experimenting with the overall toy :-)

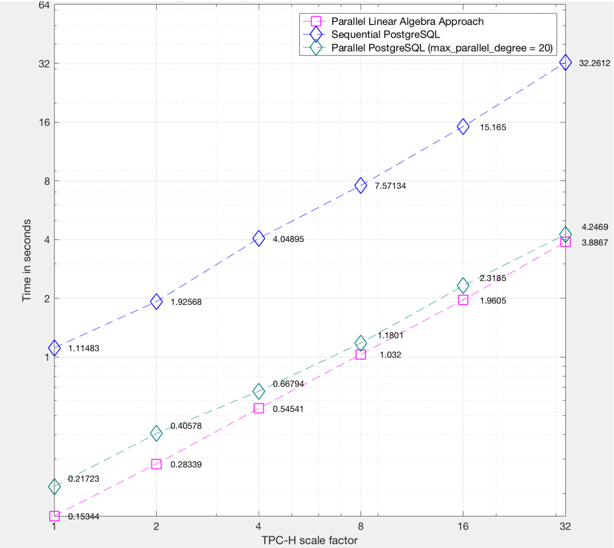
In particular,

- Compare LA paths with TPC-H query plans
- Complete the benchmark already carried out.[9]

[9] R.Pontes, Benchmarking a Linear Algebra Approach to OLAP (2015).

Preliminary results (Search6)

(Filipe Oliveira, Sérgio Caldas, MSc project on HPC)



Theory side

- Compare with related work on **columnar** DB systems
- Parametrize DSL on appropriate **semirings** for non arithmetic aggregations (*min*, *max* etc)
- Extend semantic coverage as much as possible, keeping the LA encoding such as e.g. in

  $$t_B^\circ \cdot t_B = id$$

  expressing **UNIQUE** constraints, or **integrity constraints** such as in e.g.

  $$p_F \leqslant t_K \cdot t_K^\circ \cdot p_F$$

  (K primary key, F foreign key.)
- **Null values** ? ...

Currently formalizing the semantics of **CUBE** in linear algebra.

Appendix

What about queries without **group by**?

Query:[10]

select
sum (r_a)
from r, s
where $r\_c = s\_b$ *and*
$5 < r\_a < 20$ *and*
$40 < r\_b < 50$ *and*
$30 < s\_a < 40$;

Star diagram:

$$\begin{array}{ccccc} 1 & \xleftarrow[r^a]{} & \#r & \xrightarrow{r_b} & B \\ & & \downarrow{\scriptstyle r_c} & & \\ & & C & \xleftarrow{s_b} \#s \xrightarrow{s_a} & A \end{array}$$

Define

$$u\ i = 5 < r[i].a < 20$$
$$v\ i = 40 < r[i].b < 50$$
$$x\ j = 30 < s[j].a < 40$$

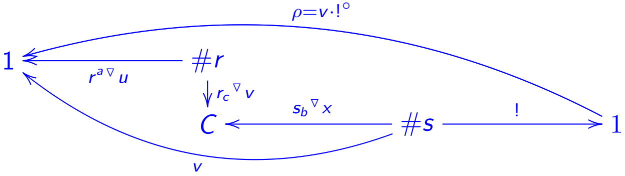
in the reduction:

$$\begin{array}{ccccc} & & 1 & & \\ & & {\scriptstyle [\![u]\!]}\uparrow & & \\ 1 & \xleftarrow[r^a]{} & \#r & \xrightarrow{[\![v]\!]} & 1 \\ & & \downarrow{\scriptstyle r_c} & & \\ & & C & \xleftarrow{s_b} \#s \xrightarrow{[\![x]\!]} & 1 \end{array}$$

[10] Example taken from D. Abadi et al, *The Design (...) Systems* (2012).

Faster, this time

Vector $\#s \xrightarrow{!} 1$ models the implicit '**group by** *all*' clause:



(21)

Thanks to (LA)

$$(M \triangledown N)^\circ \cdot (P \triangledown Q) = (M^\circ \cdot P) \times (N^\circ \cdot Q) \tag{22}$$

$$b\ (v^\circ \cdot u)\ a = v[b] \times u[a] \tag{23}$$

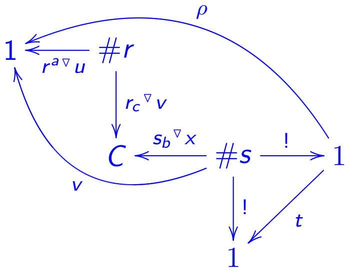
$$1\ (! \cdot M)\ a = \langle \sum\ b\ ::\ b\ M\ a \rangle \tag{24}$$

we get the expected output scalar:

$$\rho = \langle \sum\ j, i\ :\ u\ i \wedge v\ i \wedge r[i].c = s[j].b \wedge x\ j\ :\ r[i].a \rangle$$

Details

Details about the "hidden" tabulation in (21):



$$t = ! \cdot (v \triangledown id) \cdot !^{\circ}$$

$\Leftrightarrow$ { (14) }

$$t = (v \triangledown !) \cdot !^{\circ}$$

$\Leftrightarrow$ { ! is the unit of Khatri-Rao }

$$t = v \cdot !^{\circ}$$

$\Leftrightarrow$ { definition of ρ }

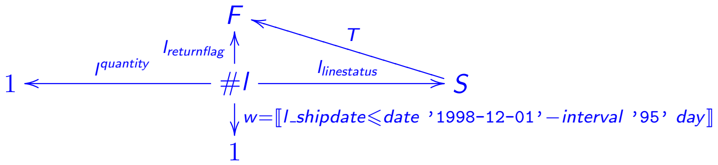
$$t = \rho$$

□

LA script for TPC-H query1 (part of)

query1 =
  select
    l_returnflag,
    l_linestatus,
    sum (*l_quantity*) *as sum_qty*,
  from
    lineitem
  where
    l_shipdate ⩽ *date* '1998-12-01' − *interval* '95' *day*
  group by
    l_returnflag,
    l_linestatus
  order by
    l_returnflag,
    l_linestatus;

LA script for TPC-H query1 (part of)



Trading between **composition** and **Khatri-Rao** (cf. performance?):

$$Q = (l_{returnflag} \triangledown l^{quantity}) \cdot (l_{linestatus} \triangledown w)^\circ$$

the same as

$$Q = (l_{returnflag} \triangledown (l^{quantity} \times w)) \cdot l^\circ_{linestatus}$$

isomorphic to

$$Q' = (l_{returnflag} \triangledown l^{quantity}) \triangledown (l_{linestatus} \triangledown w) \cdot !^\circ$$