Computing for Musicology
(Course code: F104N5)
4. Map & filter for (quantitative) musical analysis

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Recall word mappings

Recall

- the *map* operator, which we've seen being extremely useful in Haskell programming and music processing.

Recalling examples of our use of *map*:

- words, ... — eg. conversion to uppercase letters:

  $$map\ toUpper\ \texttt{"Mendelssohn"} = \texttt{"MENDELSSOHN"}$$

- music parts, ... — eg. augmentation, and so on:

  $$map\ (id \times (/n))\ p$$

  augments/diminishes part p depending on whether n is smaller or larger than 1.

Recall word filtering

Further recall filtering:

$$filter\ (\in [\texttt{'a'} \ldots \texttt{'z'}])$$
$$\texttt{"Joseph Haydn died two hundred years ago"}$$

yielding

$$\texttt{"osephydndiedtwohundredyersgo"}$$

based on the membership **property** $(\in [\texttt{'a'} \ldots \texttt{'z'}])$ which selects lowercase characters.

Filtering extends to any kind of list in Haskell, not just words, eg:

$$filter\ odd\ [1\ldots]$$

yields the list of all odd natural numbers

$$[1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, \ldots]$$

Combining map and filter

These two operations — *map* and *filter* play a major role in programming, in a way such that they complement each other:

- *filter p* **selects** those elements in a list which are of interest according to selection criterion *p*;
- *map f* **transforms** all elements in a list, one after the other, according to transformation *f*.

One can combine these two operations in a single operation using **composition**:

$$map\ f \cdot filter\ p$$

This performs a selection **followed by** a transformation. For instance,

$$((map\ toUpper) \cdot (filter\ notVowel))\ \texttt{"Joseph Haydn"}$$

yields `"JSPH HYDN"`.

Comprehending map and filter

Haskell offers an alternative notation for

(map f . filter p)l

in which we easily see **selection** and **transformation** explicitly combined:

$$[f\ c \mid c \leftarrow l, p\ c] = (map\ f \cdot filter\ p)\ l$$

Notation $[f\ c \mid c \leftarrow l, p\ c]$ means:

take those elements from l, one at a time, which satisfy p, and transform them via f.

For instance,

$$[toUpper\ c \mid c \leftarrow \texttt{"Joseph Haydn"}, notVowel\ c]$$

yields the same `"JSPH HYDN"`.

Quantitative analysis

- Maps and filters are also useful in performing **quantitative** analysis.
- Suppose, for instance, that a given list l contains all the published works of a given composer and that, for each such work w:
  - $date\ w$ yields its date of composition (eg. 1805)
  - $desc\ w$ yields the title, or description of w (eg. `"Symphony No. 3 in E flat major 'Eroica'"`)
  - $op\ w$ yields its opus number (eg. opus 55)
- Now suppose we want to count the number of works composed at a given date d.

Querying

Clearly, we have to filter list l by selecting only the works composed by such date, eg. by writing

$$[w \mid w \leftarrow l, date\ w \equiv d]$$

Then *counting* amounts to calculating the length of such a list of selections:

$$length\ [o \mid o \leftarrow l, date\ o \equiv d]$$

What we have just done is known in the literature of **information retrieval** as *querying*:

> *Given a particular source of data (list* l *in our example),* **querying** *such data source consists of obtaining information (eg. statistical) from such information.*

Querying

- As a rule, queries are to be repeated over and over again as the data source evolves (eg. as performed by the National Statistics Institute).
- It is thus a good idea to give queries a *name*, as we do below concerning the query in the previous slide:

$$nrOfWorksByDat\ d\ l = length\ [o \mid o \leftarrow l, date\ o \equiv d]$$

An alternative definition for this query involving *filter* is:

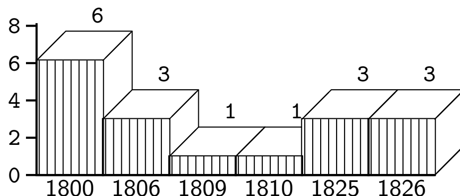
$$nrOfWorksByDat\ d = length \cdot (filter\ ((\equiv d) \cdot date))$$

Counting data and building histograms

From the Wikipedia:

> *In statistics, a* **histogram** *is a graphical display of tabulated frequencies, shown as bars. (...) In a more general (...) sense, a histogram is a mapping* m_i *that counts the number of observations that fall into various disjoint categories (known as* **bins***)*

For instance,



depicts the histogram of the number of string quartets composed per year by L. van Beethoven (1770-1827).

Counting and building histograms

In Haskell, the contents of **histograms** are simply lists of pairs,

[(b1,c1),...,(bn,cn)]

where the *b*s are *bins* and the *c*s are numbers.

For example, list

$$[(1800,6),(1806,3),(1809,1),(1810,1),(1825,3),(1826,3)]$$

contains the information depicted in the previous slide, where the bins are years 1800,1806,...,1826.

From any list one can calculate the histogram of its contents by counting how repeated each element in the list is:

$$\begin{aligned} & \mathit{hist}\ l = \mathit{nub}\,[(x, \mathit{count}\ x\ l) \mid x \leftarrow l] \\ & \quad \textbf{where}\ \mathit{count}\ a\ l = \mathit{length}\,[x \mid x \leftarrow l, x \equiv a] \end{aligned}$$

(The *nub* function eliminates repetition of pairs.)

Counting and building histograms

- Clearly, in building a histogram most of the work goes into selecting all occurrences of the data of interest in a list.
- Then *hist* yields the histogram from this list.
- Example: we want to produce the histogram of keys in Beethoven's works. For this we assume that

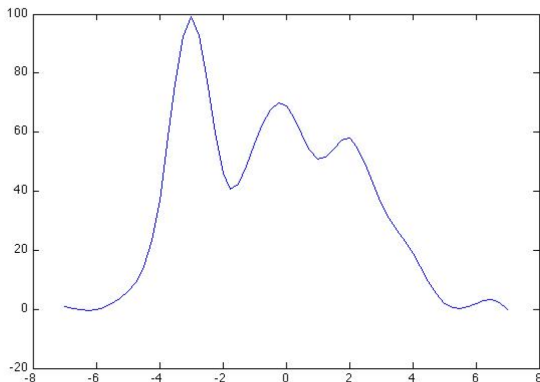
  $key\ x$

  yields the key of work x.
- Clearly, *map key* extracts the list of all keys, to be passed on to *hist*. The query to build is then:

$$keyHist = hist \cdot (map\ key)$$

Lab assignment

Perform data analysis over library `LvB.hs` and compare your results which the following plot telling the number of works written by the composer on a particular clef (minor and major modes ignored). Observe the peak at point -3, corresponding to E$\flat$ major and C minor:



Exercises

Exercise 1: In the context of the previous slides, complete the query in Haskell

$$getSymphonies\ l = ...$$

which searches data source l and finds the opus number and publishing date of each symphony (should yield the empty list in case the composer wrote none!).
Suggestion: involves checking whether word `"Symphony"` is a prefix of the description of each work. Writing auxiliary predicate

$$isSymphony\ w \equiv ...$$

will help. Load library `LvB.hs` (catalogue of woks by L. van Beethoven (1770-1827), WoO's excluded) and test your query.

Exercises

Exercise 2: Write the query which computed the *works per year* histogram of Beethoven's string quartets given above.

Exercise 3: Write the query which computes the *works per year* histogram of Beethoven's piano sonatas between 1800 and 1810.

Exercise 4: Run *keyHist* for Beethoven's violin and piano sonatas only.

Exercises

Exercise 5: Implement the function *concat* which joins a list of lists into a single list by completing and simplifying

$$concat\ [\,] = \ldots$$
$$concat\ [a] = \ldots$$
$$concat\ (l + r) = \ldots$$

Implementing *filter*

Let us try and define *filter p* ourselves. As earlier on, three properties of the function being defined need to be identified:

$$\begin{aligned}
&\mathit{filter}\ p\ [\,] = \ldots. \\
&\mathit{filter}\ p\ [c] = \ldots. \\
&\mathit{filter}\ p\ (w +\!\!+\ y) = \ldots\ (\mathit{filter}\ p\ w)\ \ldots\ (\mathit{filter}\ p\ y)\ \ldots
\end{aligned}$$

The first and last aren't too difficult to find:

$$\begin{aligned}
&\mathit{filter}\ p\ [\,] = [\,] \\
&\mathit{filter}\ p\ [c] = \ldots. \\
&\mathit{filter}\ p\ (w +\!\!+\ y) = (\mathit{filter}\ p\ w) +\!\!+\ (\mathit{filter}\ p\ y)
\end{aligned}$$

Implementing *filter*

The second requires testing whether c fulfills selection criterion (property) p. Haskell offers special syntax for this, either

$$\begin{array}{l} \mathit{filter}\ p\ [c] \\ \quad \mid p\ c = [c] \\ \quad \mid \mathit{otherwise} = [\,] \end{array}$$

or its inlined version:

$$\mathit{filter}\ p\ [c] = \textbf{if}\ p\ c\ \textbf{then}\ [c]\ \textbf{else}\ [\,]$$

Implementing *filter*

To complete the encoding, we incorporate the middle clause into the third by instantiating w to $[c]$ and simplifying:

$$
\begin{array}{ll}
 & \mathit{filter}\ p\ ([c] +\!\!+\ y) = (\mathit{filter}\ p\ [c]) +\!\!+ (\mathit{filter}\ p\ y) \\
\equiv & \qquad \{\ \text{simplification of left hand side}\ \} \\
 & \mathit{filter}\ p\ (c : y) = (\mathit{filter}\ p\ [c]) +\!\!+ (\mathit{filter}\ p\ y) \\
\equiv & \qquad \{\ \text{substitution in right hand side}\ \} \\
 & \mathit{filter}\ p\ (c : y) = (\textbf{if}\ p\ c\ \textbf{then}\ [c]\ \textbf{else}\ [\,]) +\!\!+ (\mathit{filter}\ p\ y)
\end{array}
$$

Implementing *filter*

Putting everything together, we obtain the following piece of Haskell:

$$\begin{aligned}&\mathit{filter}\ p\ [\,] = [\,]\\ &\mathit{filter}\ p\ (c : y) = (\textbf{if}\ p\ c\ \textbf{then}\ [c]\ \textbf{else}\ [\,]) +\!\!+ (\mathit{filter}\ p\ y)\end{aligned}$$

By doing a similar exercise one obtains the following Haskell for *map f*:

$$\begin{aligned}&\mathit{map}\ f\ [\,] = [\,]\\ &\mathit{map}\ f\ (c : y) = [f\ c] +\!\!+ (\mathit{map}\ f\ y)\end{aligned}$$

which simplifies to:

$$\begin{aligned}&\mathit{map}\ f\ [\,] = [\,]\\ &\mathit{map}\ f\ (c : y) = (f\ c) : (\mathit{map}\ f\ y)\end{aligned}$$

A glimpse at multi-dimensional analysis

(Not included in the current version of these slides)

More about Haskell

If you want to know more about Haskell (including its application to music synthesis) have a look at the following (really good) book:

> *P. Hudak: The Haskell School of Expression - Learning Functional Programming Through Multimedia. Cambridge University Press, 2000. ISBN 0-521-64408-9.*